Efficient Simulation of Interconnect Networks With Frequency-Dependent Lossy Transmission Lines

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***Abstract*—Accurate and efficient moment generation methods are important when using moment matching based circuit reduction techniques for the simulation of transients on transmission line networks. In this paper, a modified matrix exponential method is presented for fast computation of the moments associated with frequency-dependent lossy transmission lines. Numerical examples are given which demonstrate the improved performance of the proposed method with respect to existing techniques.**

***Index Terms*—Circuit reduction, circuit simulation, matrix exponential method, moment matching, transmission lines.**

I. Introduction

MODEL-REDUCTION techniques have been successfully used for the rapid simulation of very large linear networks. A linear system can be reduced using moment matching techniques based on Padé approximation methods, such as Asymptotic Waveform Evaluation [1]. When applying moment matching techniques to simulation of interconnect networks containing lossy coupled transmission lines, the transmission line moments can be generated by using either the eigenvalue moment method [2] or the matrix exponential method [3]. In the eigenvalue moment method, moment generation is performed using the eigenvalues and eigenvectors of the transmission line propagation matrix. The truncation error in the computation of the eigenvalues and eigenvectors increases for higher order moments, resulting in a degradation of accuracy for waveform evaluation. The matrix exponential method was introduced to improve the computational accuracy of the moments. The method generates moments by expanding the transmission line parameter matrix as a Taylor series. In [4], the matrix exponential method was extended to handle transmission lines with frequency-dependent parameters. Although the matrix exponential method yields more accurate moments, it is computationally more costly than the eigenvalue method due to the slow convergence of the matrix exponential series. In this paper, we propose a modified matrix exponential method for generating the moments of frequency-dependent lossy transmission lines. The proposed method yields the same accuracy as the original matrix exponential method, but also has a computational efficiency which is comparable to that of the eigenvalue moment method.

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II. Formulation

Consider a linear interconnect network containing distributed transmission lines and its model using a modified nodal admittance formulation [5]. When employing the moment matching approach, the generation of frequency derivatives (moments) of the modified nodal admittance matrix is required, which in turn requires computation of the transmission line moments. In the case of coupled lossy transmission lines with frequency-dependent parameters, the transmission line moments can be derived from the Laplace-domain partial differential equations

$$\frac{\partial}{\partial x}\begin{bmatrix}\mathbf{V}(x,s)\\ \mathbf{I}(x,s)\end{bmatrix} = \begin{bmatrix}\mathbf{0} & -\mathbf{Z}(s)\\ -\mathbf{Y}(s) & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}(x,s)\\ \mathbf{I}(x,s)\end{bmatrix} \quad (1)$$

with

$$\mathbf{Z}(s) = \mathbf{R}(s) + s\mathbf{L}(s) \quad (2)$$

$$\mathbf{Y}(s) = \mathbf{G}(s) + s\mathbf{C}(s) \quad (3)$$

where $\mathbf{V}(x,s)$ and $\mathbf{I}(x,s)$ are vectors representing the line voltages and the line currents, respectively, at points along the transmission line, $\mathbf{Z}(s)$ and $\mathbf{Y}(s)$ are the per-unit-length (p.u.l.) impedance and admittance matrices, and $\mathbf{R}(s)$, $\mathbf{G}(s)$, $\mathbf{L}(s)$, and $\mathbf{C}(s)$ are the p.u.l. frequency-dependent series resistance, shunt conductance, series inductance, and shunt capacitance transmission line parameter matrices, respectively.

From (1)–(3), the relationship between the voltages and currents at the far end of a transmission line of length d, $\mathbf{V}(d,s)$, and $\mathbf{I}(d,s)$, and those at the near end, $\mathbf{V}(0,s)$ and $\mathbf{I}(0,s)$, can be described in terms of the transmission line parameter matrix $\mathbf{T}$ as

$$\begin{bmatrix}\mathbf{V}(d,s)\\ \mathbf{I}(d,s)\end{bmatrix} = \mathbf{T}(s)\begin{bmatrix}\mathbf{V}(0,s)\\ \mathbf{I}(0,s)\end{bmatrix} \quad (4)$$

where

$$\mathbf{T}(s) = e^{\mathbf{F}(s)d}, \quad \mathbf{F}(s) = \begin{bmatrix}\mathbf{0} & -\mathbf{Z}(s)\\ -\mathbf{Y}(s) & \mathbf{0}\end{bmatrix}. \quad (5)$$

The p.u.l. impedance and admittance $\mathbf{Z}(s)$ and $\mathbf{Y}(s)$ matrices can be expanded as a Taylor series in the form

$$\mathbf{Z}(s) = \sum_{n=0}^{\infty}\mathbf{Z}_n s^n, \quad \mathbf{Y}(s) = \sum_{n=0}^{\infty}\mathbf{Y}_n s^n \quad (6)$$

and, thus

$$\mathbf{F}(s) = \sum_{n=0}^{\infty}\mathbf{F}_n s^n, \quad \mathbf{F}_n = \begin{bmatrix}\mathbf{0} & -\mathbf{Z}_n\\ -\mathbf{Y}_n & \mathbf{0}\end{bmatrix}. \quad (7)$$

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For the special case when the line parameters $\mathbf{R}$, $\mathbf{G}$, $\mathbf{L}$ and $\mathbf{C}$ are frequency independent, we have $\mathbf{Z}_0 = \mathbf{R}$, $\mathbf{Z}_1 = \mathbf{L}$, $\mathbf{Y}_0 = \mathbf{G}$, $\mathbf{Y}_1 = \mathbf{C}$, and $\mathbf{Z}_n = \mathbf{Y}_n = \mathbf{0}$ for $n > 1$.

In the implementation of moment matching techniques, the coefficients of the Taylor series expansion of the transmission matrix $\mathbf{T}(s)$ are required

$$\mathbf{T}(s) = [\mathbf{1}] + \frac{\mathbf{F}(s)d}{1!} + \frac{\mathbf{F}^2(s)d^2}{2!} + \cdots + \frac{\mathbf{F}^n(s)d^n}{n!} + \cdots. \quad (8)$$

In the matrix exponential method presented in [3], [4], the expansion coefficients are obtained by substituting (7) into (8) and collecting terms with same power of s. The computation of the coefficients in this manner usually requires summation of a large number of terms due to the slow convergence of a series of the exponential type.

In order to more efficiently generate the moments of transmission line systems, a modified matrix exponential method is presented. Starting from (6), the product $\mathbf{Z}(s)\mathbf{Y}(s)$ can be written as a Taylor series expansion in the form

$$\mathbf{Z}(s)\mathbf{Y}(s) = \sum_{n=0}^{\infty} \mathbf{W}_n s^n, \quad \mathbf{W}_n = \sum_{i=0}^{n} \mathbf{Z}_{n-i}\mathbf{Y}_i. \quad (9)$$

Let the matrix $\mathbf{T}(s)$ be expanded in the form

$$\mathbf{T}(s) = \begin{bmatrix} \mathbf{A}(s) & \mathbf{B}(s) \\ \mathbf{C}(s) & \mathbf{D}(s) \end{bmatrix} = \sum_{n=0}^{\infty} \begin{bmatrix} \mathbf{A}_n & \mathbf{B}_n \\ \mathbf{C}_n & \mathbf{D}_n \end{bmatrix} s^n. \quad (10)$$

By expanding (5) in an exponential series as in (8), we have [see (11) at the bottom of the page]. Equating (10) and (11) gives

$$\mathbf{A}(s) = \sum_{n=0}^{\infty} \mathbf{A}_n s^n = [1] + \frac{d^2}{2!}\mathbf{Z}(s)\mathbf{Y}(s) + \frac{d^4}{4!}\mathbf{Z}(s)\mathbf{Y}(s)\mathbf{Z}(s)\mathbf{Y}(s) + \cdots \quad (12)$$

Now, using (9) and matching powers of s yields the recursive formulas

$$\mathbf{A}_0 = [\mathbf{1}] + \sum_{j=1}^{\infty} \mathbf{A}_{1,j}, \quad \mathbf{A}_i = \sum_{j=1}^{\infty} \mathbf{A}_{i+1,j}; \quad i = 1, 2, \ldots \quad (13)$$

where

$$\mathbf{A}_{i,1} = \frac{d^2\mathbf{W}_{i-1}}{2}, \quad \mathbf{A}_{i,j} = d^2 \sum_{k=0}^{i-1} \frac{\mathbf{W}_k \mathbf{A}_{i-k,j-1}}{2j(2j-1)}, \quad j = 2, 3, \ldots. \quad (14)$$

It can be easily shown that $\mathbf{B}_n$, $\mathbf{C}_n$, and $\mathbf{D}_n$, for $n \geq 0$, are also calculated as

$$\mathbf{D}_n = \mathbf{A}_n^t \quad (15)$$

$$\mathbf{C}_n = -d\left[\mathbf{Y}_n - \sum_{k=0}^{n} \mathbf{Y}_k \mathbf{U}_{n-k}\right] \quad (16)$$

$$\mathbf{B}_n = -d\left[\mathbf{Z}_n - \sum_{k=0}^{n} \mathbf{Z}_k \mathbf{U}_{n-k}^t\right] \quad (17)$$

where the superscript t denotes the transpose of a matrix and

$$\mathbf{U}_i = \sum_{j=1}^{\infty} \frac{\mathbf{A}_{i+1,j}}{2j+1}, \quad i = 0, 1, \ldots. \quad (18)$$

It can be seen that the number of terms in $\mathbf{U}_i$ required for a desired accuracy is the same as in $\mathbf{A}_i$ [see (13)]. Compared to the original matrix exponential method, it has been found that the proposed approach is more efficient in that the series in (13) requires a relatively smaller number of terms for convergence. The faster convergence is due to two factors. First, in contrast to (8), the terms in the series (12) will decay more quickly due to the fast increase in the values of the denominators. Secondly, the size of the matrices in (12) is only half that of the matrices in (8). This results in less computational effort for calculating the line moments by the proposed recursive procedure.

III. Numerical Examples

Two examples are given to demonstrate the performance of the proposed method. As a first example, consider the interconnect circuit from [3] as shown in Fig. 1, which consists of two cascaded lossy transmission lines with identical length d. Line 1 is characterized by the parameters $Rd = 25\,\Omega$, $Gd = 0.005$ S, $Ld = 0.1\,\mu$H, and $Cd = 40$ pF. Line 2 has the same parameters except $Rd = 3.125\,\Omega$. Fig. 2 displays the magnitude of the moments of the voltage V_{out} at a frequency expansion point about the origin as generated by the proposed method and by the matrix exponential method [3]. Identical results are obtained, but with less computational effort. For example, for line 1 and using the smallest machine precision as the truncation criterion, the matrix exponential method (8) typically required twice as many terms for convergence as compared to the proposed method (12) (28 versus 14 for moment number 15 in Fig. 2). Fig. 2 also shows that the moments generated by the eigenvalue moment method exhibit an increase in numerical truncation error for higher order moments.

In a second example, a printed circuit coplanar strips transmission line is considered with the skin effect loss taken

$$\mathbf{T}(s) = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} + \frac{d}{1!}\begin{bmatrix} \mathbf{0} & -\mathbf{Z}(s) \\ -\mathbf{Y}(s) & \mathbf{0} \end{bmatrix} + \frac{d^2}{2!}\begin{bmatrix} \mathbf{Z}(s)\mathbf{Y}(s) & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}(s)\mathbf{Z}(s) \end{bmatrix} + \frac{d^3}{3!}\begin{bmatrix} \mathbf{0} & -\mathbf{Z}(s)\mathbf{Y}(s)\mathbf{Z}(s) \\ -\mathbf{Y}(s)\mathbf{Z}(s)\mathbf{Y}(s) & \mathbf{0} \end{bmatrix} + \frac{d^4}{4!}\begin{bmatrix} \mathbf{Z}(s)\mathbf{Y}(s)\mathbf{Z}(s)\mathbf{Y}(s) & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}(s)\mathbf{Z}(s)\mathbf{Y}(s)\mathbf{Z}(s) \end{bmatrix} + \cdots \quad (11)$$

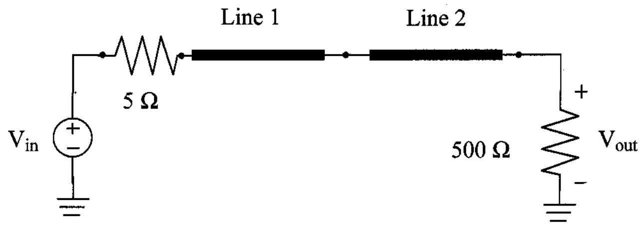

Fig. 1. Circuit containing two cascaded lossy transmission lines (from [3]).

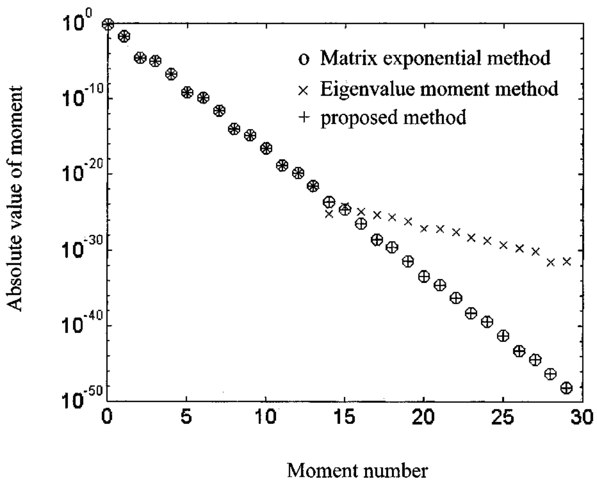

Fig. 2. Comparison of moments generated by the proposed technique, the matrix exponential method, and the eigenvalue moment method.

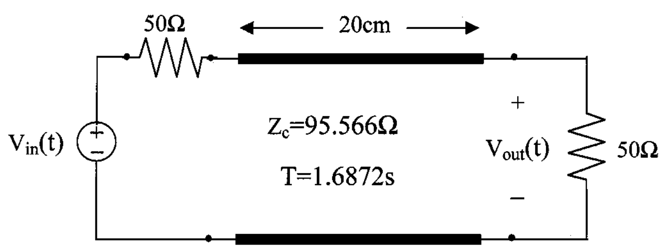

Fig. 3. Lossy printed circuit transmission line (from [6]).

into account [6]. The circuit is shown in Fig. 3 and has a p.u.l. inductance $L = 0.805\,969$ μH/m and capacitance $C = 88.2488$ pF/m. A frequency selective conductor impedance Z_i, due to the skin effect, is accounted for as [6] where

$$Z_i = \begin{cases} R_{dc}\left(1 + \frac{jf}{f_0}\right) & f \leq f_0 \\ R_{dc}\sqrt{\frac{f}{f_0}}(1+j) & f \geq f_0 \end{cases}$$

where $R_{dc} = 86.207\ \Omega/\text{m}$ is the p.u.l. dc resistance and $f_0 = 393.06$ MHz. The source voltage is a ramp function rising to 1V in 50 ps. The transient response at the load resistor is shown in Fig. 4, as computed, using the moment matching simulation technique described in [7], where the line was divided into four identical sections. The moments for 9 frequency expansion points, with $f_{\max} = 6.5$ GHz, were generated using both the matrix exponential method [3], [4] and the proposed method. Fig. 4 shows the results are in good agreement. The number of terms required to achieve convergence using the proposed method was again half that required by the matrix exponential method.

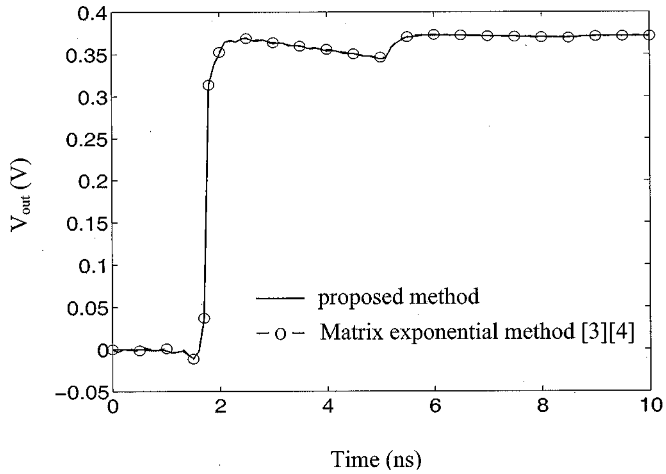

Fig. 4. Transient response at the load end of the circuit in Fig. 3.

IV. Conclusion

A modified matrix exponential method has been proposed for generating the moments of coupled lossy frequency-dependent transmission lines. Numerical results indicate that the proposed method yields both accurate moments and reduced computational cost when compared to the eigenvalue method or matrix exponential method.

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